DOCUMENT RESUME

ED 203 239 CG 015 222

AUTHOR Feibel, Werner
TITLE Representation and Its Role in Understanding Problem Solving.
PUB DATE Sep 80
NOTE 17p.: Paper presented at the Annual Convention of the American Psychological Association (88th, Montreal, Quebec, Canada, September 1-5, 1980).

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Abstract Reasoning: Association (Psychology): *Classification: *Cognitive Processes: Cognitive Style: *Concept Formation: Foreign Countries: Individual Differences: *Logical Thinking: *Problem Solving
IDENTIFIERS *West Germany

ABSTRACT

Information about individuals' representations of problems was obtained from studies on reasoning and problem categorization. College students (N=62) participated in a study of the transition from concrete to formal reasoning using the pendulum and chemical yellow tasks. Over a 10-12 week period, 47 students participated in 10 training sessions on various types of problems. Concrete reasoners were more likely than transitional reasoners to represent a conditional statement as equivalent to its converse in sorites and implication problems. Transitional reasoners were more likely than concrete reasoners to mention structural aspects of problems as common characteristics of problems. Results suggest that consideration of individuals' representations of problems can enhance descriptions of structural aspects of reasoning and account for problem-solving behavior on certain logic tasks. College students (N=17) participating in a pilot study on problem categorization sorted 1 of 2 partially identical sets of 55 problems into groups on the basis of similarity and dis- cussed their categorization and their reasons for sorting. Results indicated that initial categorizations of problems rely on relatively superficial, specific, and easily perceivable problem characteristics rather than structural characteristics which are geared more directly toward solution. (NRB)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED203239

REPRESENTATION AND ITS ROLE IN UNDERSTANDING PROBLEM SOLVING

Werner Feibel
Psychologisches Institut
Universität Regensburg
Regensburg, West Germany

U S DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

Werner Feibel

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

CG 015222

Paper presented at the Annual Convention of the American Psychological Association, Montreal, Canada, September 1-5, 1980.

ABSTRACT

TWO SETS OF DATA PROVIDE INFORMATION ABOUT INDIVIDUALS' REPRESENTATIONS OF PROBLEMS, AND INDICATE THE VALUE OF INVESTIGATING THE ROLE OF REPRESENTATION IN REASONING AND PROBLEM SOLVING.

I. SIXTY-TWO COLLEGE STUDENTS PARTICIPATED IN A STUDY ON THE TRANSITION FROM CONCRETE TO FORMAL REASONING. LEVEL OF REASONING WAS ASSESSED USING THE PENDULUM AND CHEMICAL YELLOW TASKS. OVER A 10-12 WEEK PERIOD, FORTY-SEVEN OF THESE STUDENTS EACH PARTICIPATED IN 10 SESSIONS DURING WHICH THEY WORKED WITH AND RECEIVED TRAINING ON VARIOUS TYPES OF PROBLEMS. REPRESENTATIONAL ASPECTS OF INDIVIDUALS' APPROACHES TO, CONCEPTIONS AND SOLUTIONS OF CERTAIN PROBLEMS ARE DISCUSSED IN RELATION TO LEVEL OF REASONING.

II. SEVENTEEN COLLEGE STUDENTS TOOK PART IN A PILOT STUDY ON PROBLEM CATEGORIZATION. INDIVIDUALS WERE GIVEN ONE OF TWO PARTIALLY IDENTICAL SETS OF 55 PROBLEMS EACH, AND WERE ASKED TO SORT THE PROBLEMS INTO GROUPS ON THE BASIS OF SIMILARITY, WHICH WAS LEFT UNDEFINED. THE STUDENTS WERE THEN ASKED ABOUT THEIR CATEGORIES AND THEIR REASONS FOR SORTING AS THEY DID. PROBLEMS WERE SELECTED IN PART FROM THE RESEARCH LITERATURE: INCLUDING TOWER OF HANOI, MISSIONARIES & CANNIBALS, CLASS INCLUSION, SYLLOGISM, AND IMPLICATION PROBLEMS. ISOMORPHS OF THESE PROBLEMS WERE ALSO INCLUDED. PROBLEM CLUSTERS WERE IDENTIFIED USING A SIMILARITY INDEX, WITH THE CRITERION BEING THAT OVER 60% OF THE INDIVIDUALS WORKING WITH THAT PROBLEM SET HAD TO GROUP THE PROBLEMS FOR THEM TO BE INCLUDED IN THE SAME CLUSTER. DATA ARE PRESENTED CONCERNING: A) THE TYPES OF CATEGORIES FORMED; AND B) THE CATEGORIZATION OF PROBLEM ISOMORPHS. A GRAIN OF SALT IS INCLUDED TO EMPHASIZE THE TENTATIVE NATURE OF THESE PILOT DATA.

AT THE END OF EACH BODY OF DATA, QUESTIONS FOR FURTHER INVESTIGATION ARE INCLUDED.

STUDY 1

A) Comparisons based on pretest assessment

1) Concrete reasoners were much more likely to represent a conditional statement as equivalent to its converse in sorites (see Fig. 1) and implication (Johnson-Laird & Wason, 1970) problems than were transitional reasoners. This equivalence was indicated by the individuals' conclusions and supported by their justification of the conclusions.

TABLES 1 & 2

2) When interviewed, after session 5, about the problems with which they had worked till then, transitional reasoners were more likely to mention structural aspects of problems (e.g., the if-then nature; working with conditions to draw conclusions) as common characteristics of problems than were concrete reasoners. The latter were more likely to mention directly perceivable aspects, such as form of presentation (i.e., pictorial or verbal) or problem content.

TABLE 3

3) When asked about their approaches to the various problems, concrete reasoners mentioned content and situation of the problem, or reiterated the problem directions; transitional reasoners mentioned the need to discount content and preconceptions while solving certain of the problems, and discussed their aprroaches in terms of general strategies.

TABLE 4 (see also TABLE 5)

NOTE: It may be useful to regard the above and subsequent tables as bearing on three different, but related, aspects of representation:

A) the representation of objects/information (tables 1,2,6)

B) individuals' use and coordination of information thus represented (tables 3,7)

C) individuals' understanding of certain aspects of the problem and their representation of it (tables 4,8)

FIGURE 1

In the sorites problems (Carroll, 1976), the person is presented with three statements and asked to determine whether each of several other statements follows from the given statements and the assumption that they are all true. Among the statements to be evaluated were converses of the original statements.

For example,

Given:
- a1) Babies are illogical
- a2) No one is despised who can manage a crocodile
- a3) Illogical persons are despised

Evaluate: A) People who are not despised can manage a crocodile

Given:
- b1) If someone reads the Times they are well-educated
- b2) No hedgehogs can read
- b3) Those who cannot read are not well educated

Evaluate: B) If someone is well educated they read the Times

TABLES 1 & 2

TABLE 1	LEVEL OF REASONING		
NUMBER OF "EQUIVALENCE" RESPONSES	CONCRETE	TRANSITIONAL	
1 OR 2	13	5	
ZERO	11	18	
TOTALS	24	23	47 = N^A

$z^2 = 5.24$ $p < .05$

TABLE 1. FREQUENCY OF EQUIVALENCE AND NON-EQUIVALENCE OF CONVERSE RESPONSES BY CONCRETE AND TRANSITIONAL REASONERS.

A: NOTE THAT N=47 HERE, BECAUSE THE PRE-POSTTEST CONTROL GROUP (N=15) DID NOT PARTICIPATE IN ANY OF THE INTERMEDIATE SESSIONS.

TABLE 2	LEVEL OF REASONING		
INFORMATION ABOUT CONVERSE	CONCRETE	TRANSITIONAL	
NECESSARY	22	11	
NOT NECESSARY	2	12	
TOTALS	24	23	47 = N

$z^2 = 10.8$ $p < .002$

TABLE 2. NUMBER OF CONCRETE AND TRANSITIONAL REASONERS REGARDING INFORMATION ABOUT CONVERSE AS NECESSARY TO EVALUATE AN IMPLICATION.

TABLES 3 & 4

TABLE 3	LEVEL OF REASONING		
COMMON CHARACTERISTICS	CONCRETE	TRANSITIONAL	
STRUCTURAL	9	17	
NON-STRUCTURAL	15	6	
TOTALS	24	23	47 = N

$z^2 = 6.30$ $p < .02$

TABLE 3. NUMBER OF PEOPLE MENTIONING STRUCTURAL OR NON-STRUCTURAL ASPECTS AS COMMON TO A SERIES OF PROBLEMS.

TABLE 4	LEVEL OF REASONING		
APPROACH ASPECTS	CONCRETE	TRANSITIONAL	
SPECIFIC	17	7	
GENERAL	7	16	
TOTALS	24	23	47 = N

$z^2 = 7.67$ $p < .006$

TABLE 4. NUMBER OF PEOPLE MENTIONING CONTENT/SPECIFIC PROBLEM DETAILS VERSUS GENERAL ASPECTS IN DISCUSSING THEIR APPROACHES TO PROBLEMS.

TABLE 5

Concrete reasoners

1. "The answers are always true, false, or irrelevant. ... Also, it seems like there are always four problems each time."

2. "The pendulum was most different. All of the problems were true, false, or irrelevant. Most of them were true-false questions; but the pendulum was different. You had to do physical manipulation and you used more observations."

3. "All of the problems were not simple, but some of them were easier to work through. The chemicals were easier than the plants."

4. "(They were) all logic problems. Some of them were mathematical. Those were difficult since I have trouble with math."

5. "...can't really say. The conclusion problems seemed similar, but they seemed different from the others."

Transitional reasoners

1. "(They all required) logic, common sense. You had to put together what facts were directly active in the statement and throw aside the rest. You had to be choosy in what facts you use, otherwise you'd get useless stuff. The problems give facts and ask for conclusions."

2. "You have to cross out (i.e., eliminate) possibilities and see what's left. ... Sometimes the symbolism was more complex. In the alligator problem, the statements were so off the wall. You have to be 'acculturated'. Otherwise, accepting the statements was hard."

3. "(The problems all involved) being able to manipulate mental statements with or without regard to reality."

4. "I broke them down to if-then problems. You had to determine what the given is and process it correctly."

5. "Some problems, such as the plants, involved trends, and some involved logical progressions."

Table 5. Examples of concrete and transitional reasoners' conceptions of and approaches to problems.

B) COMPARISONS BASED ON POSTTEST ASSESSMENT

A COMPARISON OF FORMAL AND PREFORMAL REASONERS WORKING WITH THE FLOATING BODIES PROBLEM REVEALED THE FOLLOWING DIFFERENCES:

1) PREFORMAL (PREDOMINANTLY TRANSITIONAL) REASONERS WERE MORE LIKELY TO ACCOUNT FOR AN OBJECT'S BEHAVIOR IN WATER BY REFERENCE TO ABSOLUTE OR "PERCEIVABLE" PROPERTIES (E.G., HAS AIR, IS HEAVY, ETC.), WHEREAS FORMAL REASONERS WERE MORE LIKELY TO REPRESENT THE OBJECT'S BEHAVIOR IN TERMS OF RELATIONAL, OR DERIVED, CHARACTERISTICS (E.G., HEAVIER THAN WATER, DOES [NOT] BREAK SURFACE TENSION, ETC.).

TABLE 6

2) IN RESPONSE TO QUESTIONS ABOUT "WHAT KINDS OF THINGS FLOAT/SINK?"

PREFORMALS WERE MORE LIKELY TO LIST PARTICULAR TYPES OF OBJECTS OR AGAIN MENTION ABSOLUTE PROPERTIES--EITHER FOR BOTH QUESTIONS OR FOR QUESTIONS ONLY ABOUT FLOATING OBJECTS--WHEREAS FORMAL REASONERS MENTIONED DENSITY OR RELATIONAL FACTORS IN RESPONSE TO BOTH QUESTIONS. INTERESTINGLY, EVERYONE WHO MENTIONED DENSITY IN RELATION TO FLOATING OBJECTS ALSO DISCUSSED SINKING OBJECTS IN THESE TERMS, WHILE THE REVERSE WAS NOT TRUE.

TABLE 7

3) WHEN ASKED TO DEFINE DENSITY, FORMAL REASONERS EXPLICITLY MENTIONED ITS RELATIONAL ASPECT, WHEREAS HALF THE PREFORMALS PROVIDED ONLY PARTIAL DEFINITIONS (E.G., NUMBER OF ATOMS OR MOLECULES, NOT POROUS, ETC.).

TABLE 8

TABLES 6 & 7

TABLE 6	LEVEL OF REASONING		
EXPLANATION	PREFORMAL	FORMAL	
ABSOLUTE	20	8	
RELATIONAL	16	18	
TOTALS	36	26	62 = N

$z^2 = 4.12$ $p < .05$

TABLE 6. NUMBER OF PEOPLE PROVIDING ABSOLUTE OR RELATIONAL PROPERTIES AS EXPLANATION FOR AN OBJECT'S BEHAVIOR IN WATER.

TABLE 7	LEVEL OF REASONING		
SUMMARY	PREFORMAL	FORMAL	
SPECIFIC	19	7	
RELATIONAL[A]	17	19	
TOTALS	36	26	62 = N

$z^2 = 4.16$ $p < .05$

TABLE 7. NUMBER OF PEOPLE LISTING PARTICULAR OBJECTS/ABSOLUTE PROPERTIES VERSUS RELATIONAL ATTRIBUTES IN SUMMARIZING WHAT KINDS OF THINGS FLOAT/SINK.

A: TO BE INCLUDED IN THIS CATEGORY, AN INDIVIDUAL HAD TO PROVIDE RELATIONAL RESPONSES IN ANSWER TO BOTH QUESTIONS--I.E., ABOUT THE KINDS OF OBJECTS THAT FLOAT AND THOSE THAT SINK.

TABLE 8

TABLE 8	LEVEL OF REASONING		
DEFINITION	PREFORMAL	FORMAL	
PARTIAL	16	3	
RELATIONAL	16	22	
TOTALS	32	25	57 = N^A

$z^2 = 9.12$ $p < .003$

TABLE 8. NUMBER OF PEOPLE PROVIDING PARTIAL VERSUS RELATIONAL DEFINITIONS OF DENSITY.

A: RESPONSES OF 3 INDIVIDUALS TO THIS QUESTION WERE INADVERTENTLY NOT RECORDED DURING THE INTERVIEW AND COULD NOT BE UNAMBIGUOUSLY EVALUATED FROM WRITTEN NOTES; 2 INDIVIDUALS WERE NOT ASKED THE QUESTION.

C) Conclusions

1) Consideration of individuals' representations of problems can help enhance descriptions of structural aspects of reasoning.

2) It can also help account for problem solving behavior on certain logic tasks (Tables 1,2).

3) Examination of the manner in which an individual represents and compares problems tells us something about the attributes and aspects of the problems that the individual considers relevant (and presumably utilizes for changing reasoning patterns (Tables 3,6).

4) Investigation of individuals' representations of the situation and their actions within that situation provides data about individuals' reflexive abstractions (Piaget, 1975; 1977a,b), and suggests (along with Tables 3 & 6) that individuals may differ not only in their reasoning, but also in the ways that reasoning is likely to change.

D) Questions

1) What is the relationship between representation and reasoning? (See, e.g., Langer, 1974) Since reciprocal influence is likely, the question becomes one of determining:

A) Under what circumstances one aspect will influence the other;

B) The nature and effects of this influence.

2) The asymmetry in people's discussion of floating versus sinking objects suggests a greater salience for one pole of an attribute--similar to the "marking" on certain poles of semantic concepts.

A) If this speculation is borne out, how can we account for this phenomenon?

B) What role does such a phenomenon play in the development of representation and reasoning? (See Bovet et al., 1975)

C) What is the relationship between such marking in the phsyical and semantic realms?

3) The present study provides only a very general indication of individuals' selection and coordination of information and experiences.

A) What are the more specific characteristics of such coordinated structures?

B) What effect does familiarity with a content area or problem type have on such coordination--with respect to both level and nature of coordination?

C) What are the respective roles (in a qualitative sense) of the individual's general knowledge, content or domain relevant knowledge, and the situations providing the information for a person's construction and use of representations?

4) Being optimistic and assuming we will eventually find a reliable and acceptably precise way of investigating abstraction <u>processes</u> in addition to products:

A) What patterns/changes in representation can be identified during development?

B) What can we learn from such patterns/changes that may be of use in education?

C) What possible future changes in reasoning and abstraction processes can be gleaned from such information?

STUDY 2

A) CLUSTERS COMMON TO BOTH PROBLEM SETS

1) MATH PROBLEMS: GROUPED ON BASIS OF CONTENT. E.G., MATHEMATICAL PROGRESSIONS INCLUDED HERE RATHER THAN WITH LOGICAL ORDERING PROBLEMS. PROBLEMS INVOLVING VARIABLES LESS LIKELY TO BE INCLUDED THAN THOSE USING NUMBERS.

2) SYLLOGISMS & SORITES: GENERALLY LABELLED LOGIC PROBLEMS. ALSO INCLUDED 2 OF 4 SYLLOGISMS IN STORY FORM. TWO IMPLICATION PROBLEMS (JOHNSON-LAIRD & WASON, 1970) WERE NOT INCLUDED.

3) REARRANGEMENTS OF PICTORIAL ELEMENTS: INCLUDED THE 2 PICTORIAL TOWER OF HANOI ISOMORPHS, AND PROBLEMS REQUIRING CUTTING A PATTERN ACCORDING TO CERTAIN RESTRICTIONS.

4) TRANSFER PROBLEMS: INCLUDED THE MISSIONARIES & CANNIBALS ISOMORPHS, AND THE 2 VERBAL TOWER OF HANOI ISOMORPHS.

B) CLUSTERS FOUND IN ONLY ONE OF THE TWO PROBLEM SETS

1) FINDING THE NEXT ELEMENT IN A LOGICAL, NON-MATHEMATICAL PROGRESSION: DISTINGUISHED BY MOST OF THE PEOPLE WHO CLUSTERED THEM TOGETHER FROM PROGRESSIONS USING NUMBERS OR PICTURES.

2) FINDING A COMMON ELEMENT IN A SET OF WORDS, OR A WORD THAT HAS SOMETHING IN COMMON WITH A GIVEN SET OF WORDS.

3) "LOGICAL MATRIX" PROBLEMS, WHERE THE SIMULTANEOUS CONSIDERATION OF VARIOUS STATEMENTS PROVIDES AN UNAMBIGUOUS SOLUTION.

4) PICTURE ORDERING OR GROUPING: ARRANGING A SET OF PICTURES IN AN EVENT SEQUENCE, OR FINDING AN ANOMALOUS PICTURE IN A SET.

C) INDIVIDUALS' GROUPINGS OF PROBLEMS USED IN RESEARCH LITERATURE

1) MISSIONARIES & CANNIBALS GROUPED TOGETHER BY ALL INDIVIDUALS; GROUPED WITH VERBAL TOWER OF HANOI PROBLEMS BY MORE THAN 60% IN EACH GROUP; NOT GROUPED WITH WATER-JUG TYPE TRANSFER PROBLEMS.

2) CLASS INCLUSION ISOMORPHS (1 PICTORIAL, 1 VERBAL, 1 "DISGUISED") GROUPED TOGETHER BY LESS THAN 25% OF INDIVIDUALS.

3) TOWER OF HANOI INCLUDED PAIRWISE (I.E., PICTORIAL OR VERBAL) IN OTHER CLUSTERS, WITH ONLY ONE PERSON GROUPING ALL 4 ISOMORPHS TOGETHER.

4) IMPLICATION PROBLEMS GROUPED TOGETHER BY FEWER THAN HALF THE INDIVIDUALS.

D) Conclusions

1) People's initial categorizations of problems rely on relatively superficial, specific, and easily perceivable problem characteristics--e.g., form of presentation, content, specific phrasing--rather than structural characteristics which are geared more directly toward solution.

2) Intraindividual differences in categorizations of problem isomorphs limit the external validity of research investigating problem solving processes for certain types of problems, since the problem typology used there is the researcher's and may differ considerably from that of the problem solver.

3) The data are tentative, so the above conclusions are merely suggested as possibilities.

E) Questions

1) What effect do the individuals' categorizations have on their solutions of problems? Specifically, what are the relationships among an individual's a) categorization; b) representation; and c) solution of a problem?

2) What are the distinctive features in, and determinants of, an individual's problem categorization? What kinds of problem taxonomies/typologies do individuals work with? E.g., what kinds of characteristics are used to identify problems as being of a certain type?

3) What is the role of familiarity with a content area or a problem type in these categorizations?

4) What is the relationship between problem categorization and level of reasoning?

REFERENCES

Bovet, M., Barancini, C., Dami, C., & Sinclair, H. Prénotions physiques chez l'enfant. Archives de Psychologie, 1975, xliii, 47-81.

Carroll, L. The Complete Works of Lewis Carroll. New York, Viking Press, 1976.

Johnson-Laird, P. & Wason, P. A theoretical analysis of insight into a reasoning task. Cognitive Psychology, 1970, 1, 134-148.

Langer, J. Interactional aspects of cognitive organization. Cognition, 1974, 3, 9-28.

Piaget, J. L'Equilibration des structures cognitives. Paris, P.U.F., 1975.

Piaget, J. Recherches sur l'abstraction réfléchissante, 1, l'abstraction des relations logico-arithmétiques. Paris, P.U.F., 1977a.

Piaget, J. Recherches sur l'abstraction réfléchissante, 2, l'abstraction de l'ordre des relations spatiales. Paris, P.U.F., 1977b.